**Appendix C: Stochastic Calculus (Mello)**

Mello’s approach is a little different. We don’t restrict ourselves to a white-noise model, but we do strictly consider Ito-like processes. Consider a sequence of identically distributed random vector variables, ΔWj(tm), the statistics of which are unspecified. And then introduce a related vector variable Xi(tm­) defined by the recursion relation:



This is similar to the Ornstein-Uhlenbeck process, but again, we restrict ourselves to the evident case that **X**(tm) is uncorrelated with Δ**W**(tm) and generalize to the case that Δ**W** isn’t necessarily white noise.

In the limit that Δt → 0, we can write:



The derivative of a function of X would be defined similarly, just like before:



(for simplicity of notation, we’ll assume there is not explicit t dependence in F). If the numerator can be written explicitly all the better, but if not, then we can use a Taylor series.



To clean up the notation, define the differential operator, averaged over ΔW as:



And then define the part which survives the small dt limit, assuming it is well defined:



Then, proceeding immediately to the average, we can write:



where, explicitly,



And as before, a Fokker-Plank equation could be written for the probability distribution function:



We might run into a problem, however, in that there are an infinite number of terms in the D-operator. This can often happen, for instance, if <δi(X,t)>ΔW ≠ 0. In such a case, we can reorganize the perturbative series around *cumulants* of δ instead of *moments* of δ. To proceed, we’ll note the following,



As this equation is quite general, we can also apply it to the function (1+D)F, rather than just F, and so obtain:



Ultimately, we can reduce it all the way down to:



Note the LHS is an average over **X**(tm) and the RHS is over just **X**(0). Next we introduce the translation generator, P(**X**) = ln(1+D(**X,**t)). The first two terms are:



With this, we can write the previous equation as:



Now we’re in position to go to the continuum. Let’s make Δt → 0 and m → ∞, such that tm ≡ t remains fixed. Then we’ll have:



And the operator P(**X**) will go to:



Assuming the limit is well defined, let the small δt limit of P/dt be denoted Pδt(**X**). So then we have:



At this point we’ll stop to note that a Taylor series expansion of the statistic could be developed:



And sometimes this is the best one can do. But we can also develop a differential equation for the statistic. Taking a derivative w/r to t gives us:



and remembering exp{tPδt[**X**(0)]} is the translation operator, we have:



From here, a Fokker-Plank equation could be written for P(**ξ**,t), like was done before:

