**Path Integrals**

Say we have a process:



And we want to calculate the probability distribution P(X(t)), basically. We can write this in terms of a path integral. First consider the probability of going from X0 to X in time t *along a particular path* X(t) = [ξ1, ξ 2, …, ξ N = X] in discreet space. The process equations and probabilities are, discreetly:





And so, the probability of the whole path is just the product of probabilities….



Filling in the expression for the white noise probability distribution, and also a Fourier representation of the delta function, we have:



We can do the ΔW integrals…



And so we have something like



Now we could do the k(t) integral technically. We would find:



(note the most probable path is the average path, and ‘spread’ is determined by b2D, as we’d expect) but that won’t help apropos putting it in a Feynman diagram form. And anyway, we want the absolute probability of going from X0 to X, not the probability of doing so along a particular path. And this is given by:



Now let’s suppose that a(ξ,t) = aξ + Δa(ξ,t). Δa(ξ,t) is in effect, a *perturbation*. Then we can say, defining A(s,t) = (d/dt – a)δ(s-t):



This is of the continuous vector integral form, where φi(t) = [ξ(t), k(t)]. We can think of the first variable as position, and the second as ‘momentum’, or ‘speed’ or something. To work out the diagrammatic expansion, we first need the propagator, defined via:



And then the general rules for the Feynman diagrams would be to first, McLaurin expand Δa(ξ,t) and b(ξ,t) in powers of ξ I suppose, to turn them into polynomials. And then every ξm·kn term would be represented as a vertex with m - ξ legs, and n - k legs. But I think we need to express these terms as m+n integrals, and so would need to introduce some delta functions. Seems like we’d probably want to switch to the Fourier basis. Perhaps we could use MFA, etc., techniques to work out the result.

**Example**

What is the path-integral representation of P[**W**(t)]?



and I guess we could write this as:



**Example**

Let’s consider the following process:



Then we have:

