**Marginal Distributions**

Given a set of correlated variables **X** and **Y**, we’re often interested in the statistics of **X** alone. In the continuum limit, the probability distribution of the **X** variables, is, directly:



To develop an evolution equation, we would start with:



And from here we have two options. We can perform the d**X**(t) integral first, introducing the Jacobian J = |∂**ξ**/∂**X**(t)|, keeping in mind it will likely depend on **Y**(t) through d**X**(t). Then we’ll ultimately have:



Then we must expand P and J in powers of d**X**(t), out to O(dt). Once done, we’d replace all combinations of **Y**(t) in d**X**(t) and d**X**(t)2 with their average, perform the **Y**(t) integral to convert P(**ξ**,**Y**(t)) to P(**ξ**), and finally perform the d**W**(t) integral to obtain our FP equation. Another option, is to go back to that prior formula and Taylor expand δ(**ξ** – **X**(t) – d**X**(t)) out to second order. Thereupon, integrate by parts, do the d**X**(t) integral, and formally the d**W**(t) integral, to obtain:



This is as far as we can go, but at this point it is typical to make a mean field approximation and replace the moments with their **Y** averages.



To round out the possibilities, we could also start from the Kolmorgorov equation in both variables,



We’ll note that d**ξ** and d**γ** likely depend on both variables. Next we integrate both sides against ∫dμ(**γ**), and assume, for simplicity that μ(**γ**) has no explicit **γ** dependence. Requisite modifications for this case will be clear. Integrating by parts, ignoring surface terms, and grouping like terms we find:



Then we use an MFA, replacing all combinations of γ, in the integrand, with their mean values. Thereupon we may execute the ∫dμ(**γ**) integral, and obtain:



There are two salient situations in which the highlighted equations would be exactly true. One is if P(**ξ**,**γ**,t) can be written as P(**ξ**,t)P(**γ**,t), or p(**ξ**,**γ**,t) as p(**ξ**,t)p(**γ**,t). But this would seem unlikely. Another possibility is that the variable **Y** is *absolutely* determined by **X** itself (and time). This supposition puts a requirement on **Y**’s evolution. If it can be written as, say:



Then **Y** evolves deterministically to the extent that one may approximate bij = 0. And **Y**sol(**ξ**,t) would be the solution to the equation:



Note that this equation is formally equivalent to the one we get when we take the average of the former w/r to d**W**, which makes sense, as a deterministic function would evolve in the same fashion as its average. Presupposing this to be true, we’d have for instance: P(**ξ**,**γ**,t) = P(**ξ**,t)δ(**γ** – **Y**sol(**ξ**,t)), or p(**ξ**,**γ**,t) = p(**ξ**,t)δ(**γ** – **Y**sol(**ξ**,t))/μ(**Y**sol(**ξ**,t)). This would make respecitively P(**ξ**,t), p(**ξ**,t), the marginal probability distribution and probability density of **X**. Filling this into, say, one of those highlighted equations for instance, we’d end up with:



The other equations would work out in the same fashion.

**Example**

Consider the coupled differential equations in Tartar’s paper:



And suppose that we have:



Then what are the evolution equations for the matrix elements of t and r΄? Let p(**ξ**,z) be the probability distribution function of tmn, i.e., pT(**ξ**,z) = <δ(t11 – ξ­11)δ(t12 – ξ12)….δ(tNN – ξNN)>, and similarly let pR(**ξ΄**,z) = <δ(r΄11 – ξ­΄11)δ(r΄12 – ξ΄12)….δ(r΄NN – ξ΄NN)> be the probability distribution function for r΄­mn. And finally, let p(**ξ**,**ξ**΄,z) = <δ(t11 – ξ­11)δ(t12 – ξ12)….δ(tNN – ξNN)∙ δ(r΄11 – ξ΄­11)δ(r΄12 – ξ΄12)….δ(r΄NN – ξ΄NN)> be their joint probability distribution. Let’s consider how p(**ξ**,z+τ) evolves by only changing the t’s. Then we have:



At this point we need to work out the averages. So…we have:



Filling these results into the equation we’d have:



Then dividing by τ and integrating over the ξ΄ variables we’d have:



The equation is a complex linear PDE. You wonder if something could be done with it? What could be done in 1D for instance? I’m going to forego writing out the equation for R(**ξ**΄,z). Obviously the process would be similar and once the equation has been obtained and solved, the averages in the T equation could be filled in, and then the T equation could be solved. The alternative is to examine the total evolution equation for p, but since the R equation decouples, I think this is the easiest approach.